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The Mathematics of Hedging

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The Mathematics of Hedging

by

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Thesis

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Dedication

To my husband, who encourages me and supports me all the time.

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I would like to thank **Dr. Jablonowski** for his support and direction during my studies at the **EER** program; this thesis couldn't have been done without him.

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I won't have such great memory during my study in the U.S. without these people. The memory of this journey will be always on my mind.

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Abstract

The Mathematics of Hedging

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The University of Texas at Austin, 2009

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Possessing the knowledge to hedge energy price risks properly is essential and crucial for running a long-term business. In the past, many hedging instruments have been invented and widely used. By using these derivatives, decision makers reduce the price risk to a certain degree.

To apply these hedging instruments to the perfect hedging strategies correctly, it is necessary to be familiar with these tools in the first place. This work introduces the financial tools widely applied in hedging, including forward contracts, futures, swaps and options. It also introduces the hedging strategies used on energy hedging. Since individuals are creating strategies according to their unique risk appetite and collected information, this work presents three risk appetites and a method of distinguishing valuable information.

With the contribution of this thesis, future works can be done in the field that connect the information valuation and energy hedging by changing the behavior in each risk appetites' hedging ratio.

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CHAPTER 1

INTRODUCTION

Hedging activities have flourished among high energy consumers or producers, including oil and gas industry, airline, aluminum smelters, and electricity generators. For example, in the oil and gas industry, exploration and production projects are long-tenor and capital intensive. Planning and controlling the price risk within the project duration is particularly important, because the risks affect the corporation operation profoundly. Failure to utilize such risk management tools is truly disastrous for an enterprise.

Corporations are exposed to uncertainties regarding a variety of prices. Hedging refers to activities undertaken by the firm in order to mitigate the impact of these uncertainties on the value of the firm. In a perfect capital market described by Modigliani and Miller (1958), in the absence of taxes, financial distress costs, contracting costs, and information costs, , there would almost be no justification for corporations to engage in hedging, including those strategies that use derivatives. However, in reality, because of the information asymmetry, market illiquidity, and social system; corporations need to hedge to avoid further undesirable risks not only for corporation value-enhancing but also for sustainable management.

Today the modern financial services industry develops risk management via hedging strategies to manage volatile markets, especially the commodity market. To make one, collecting useful information is necessary. A widely accepted method recognizes and evaluates the information in the monetary value. Nevertheless, identical information is valued differently by individuals; this is because each individual has varying degrees of risk tolerance. Thus, it is necessary to categorize individual risk type before we want to acquire the valuable information.

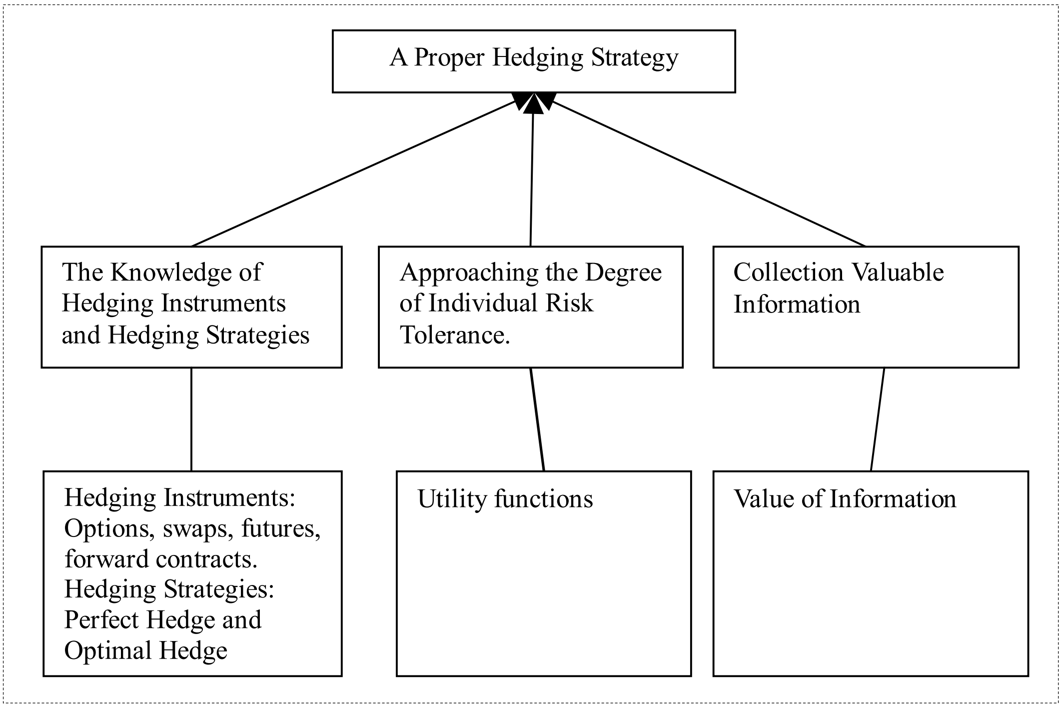


Figure 1.1 Schematic of proper hedging strategy

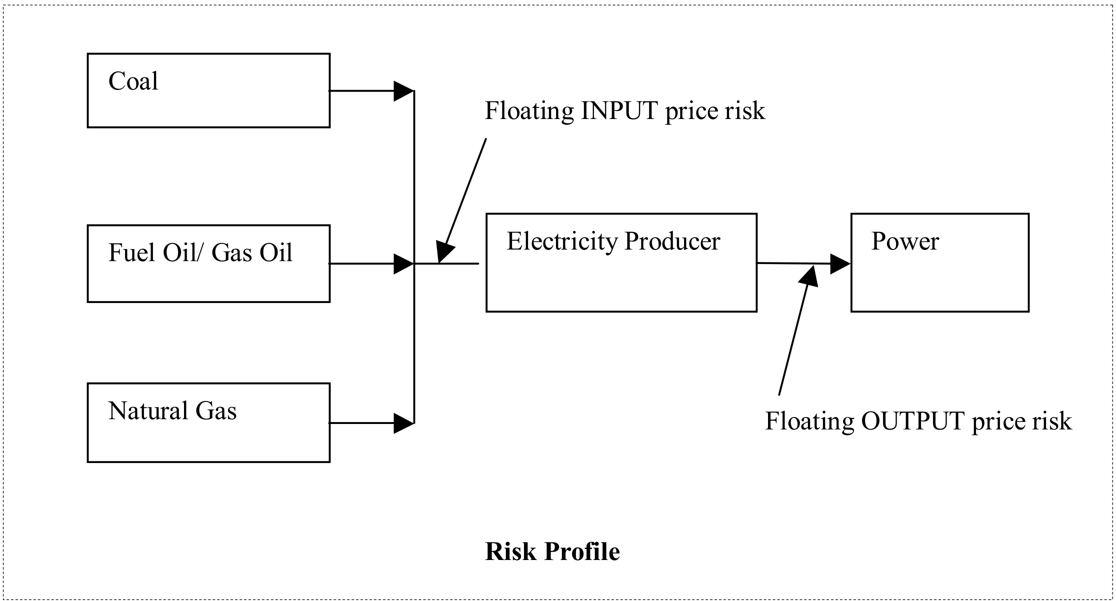


Figure 1.2 Electricity producer's risk profile.

CHAPTER 2
HEDGING INSTRUMENTS

The four main hedging tools are forwards, futures contracts, swaps, and options. And these derivatives play an important role in everyday commerce, since they provide effective tools for hedging risks involving the underlying variables. For an enterprise, the main uses of derivatives are minimizing the price risk, locking in the profits, and reducing the balance sheet volatility.

Derivatives, in broad definition, are securities whose payoffs (values) are related to other types of financial assets such as gold, crude oil, or security stocks. In practice, however, this broad definition is often restricted to securities whose payoffs are explicitly tied to the price of some other financial security. An example of such a derivative security is a certificate that can be redeemed in 6 months for an amount equal to the price, of a barrel of WTI. In fact, most real derivatives are fashioned to have important risk control features, allowing some players in a market to hedge their risk and others to take this potential profit-making opportunity to make money.

2.1 Forward Contracts

Forward contracts are simpler than, but closely related to, futures; it is a financial instrument which is a legal document, the terms of which bind the two parties involved to a specific transaction in the future.

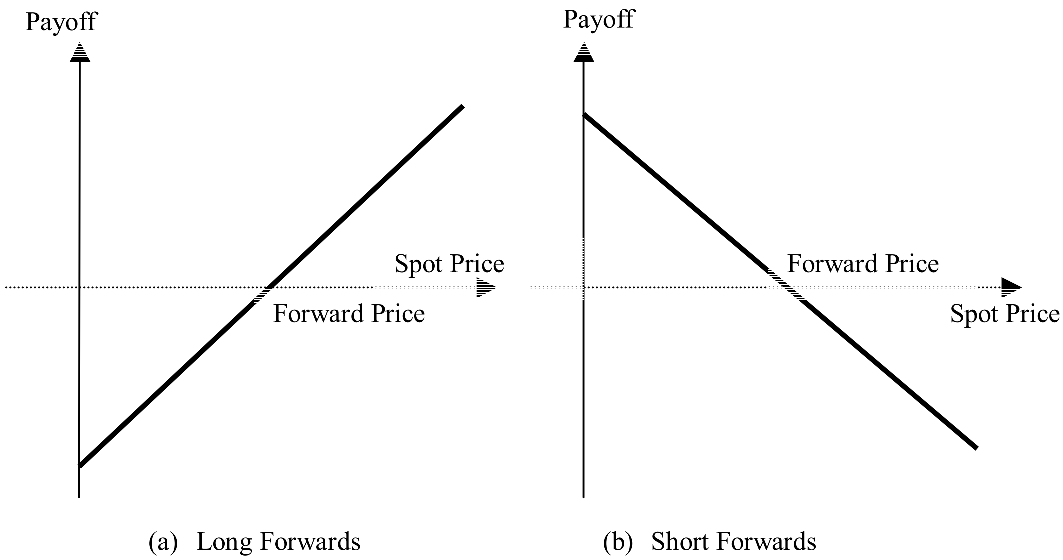


Figure 2.1 Payoff of a forward contract (a) Long forwards (b) Short forward

2.1.1 Basic of Forward Contract

Forward contracts have been extended in modern times to include underlying assets other than physical commodities. For example, many corporations use forward contracts on foreign currency, equities, interest rates, or commodities. Forward contracts for commodities have existed for thousands of years, for they are indeed a direct adjunct to commerce. For both a commodity buying side and selling side, doing hedge is to lock the price risk. When a forward contract is based on a commodity, it means the contract holder has the obligation to purchase or to sell the specific amount of a commodity at a specific price and at a specific time in the future. Every contract must have two parties, a buyer and a seller. A realistic example is an electricity company uses a forward contract to purchase 2,500 MMBtu of natural gas at 3.67 dollars per MMBtu on the 15th of March next year. In this case, there is no reference to a payoff, because there is only an obligation guaranteeing the purchase of natural gas. In fact the payoff is implied; it will be determined by the price of natural gas in March next year. At the expiration day, if the price of natural gas is \$4.67 per MMBtu, the contract would have \$2500 profit from gaining \$1.0 per MMBtu, and 2500 MMBtu per contract. On the other hand, if the price goes down to \$2.67 per

MMBtu, the contract holder will lose $2500 per contract.

Most forward contracts specify that all claims be settled at the defined future date (or dates); both parties must carry out their side of the agreement at that time. Almost always, the initial payment associated with a forward contract is zero; neither party pays any money to obtain the contract (although a security deposit is sometimes required of both parties). The forward price is the price that applies at delivery. This price is negotiated so that the initial payment is zero; that is, the value of the forward contract is zero when it is initiated.

Opposite the forward market, an open market for immediate delivery of the underlying asset, is the spot market, because the forward market always trades contracts for future delivery. Since the spot market price may fluctuate during the period of a forward contract, although the initial value of a forward contract is zero, its later value will vary as a function of the spot price of the underlying asset (or assets).

2.1.2 Two Prices Associate with Forward Contracts

There are two prices or values associated with a forward contract. The first is the forward price *F*, specified when a forward contract is written, which is the delivery price of a unit of the underlying asset to be delivered at a specific future date. The second is the current value of the forward contract denoted by *f*. Initially there should be no money exchanged when a contract agreement is completed; the forward price *F* is determined at time 0 at $f = 0$. However, the value *f* may change from time to time; it depends on variations of the spot price of the underlying asset.

Before we analyze the theoretical initial forward price *F*, some assumptions should be made such as no transaction costs, no storage cost, availability for short selling, and arbitrary division of assets. Based on the assumptions, equality exists between the spot price and the forward price. It is assumed that the spot price of the underlying is *S* and the price of a forward contract delivering at *T* is *F*. One can sell one 1,000 bbl crude oil forward contract at *F* per barrel and buy 1,000 bbls crude oil at *S* per bbl from the spot market simultaneously. In this transaction, cash flows is (-1,000*S*, 1,000*F*), because one needs to pay in the spot market at time 0 and receive the *F* at time *T*. One can reserve the crude initially to sell it at time *T* and assume $d(0, T)$ is the discount factor between 0 and *T*. Thus, we can assert the relation between *S* and *F* is $S = d(0, T) F$, and when we use continuous compounded interest rate the discount factor becomes e^{rT}, and $F = Se^{rT}$.

Under the previous assumptions, no one can arbitrage via the operations; therefore, the netting cash flow must be zero. Otherwise, anyone can earn extra money from short selling in the spot market or buying in the spot market with the opposite action with a forward contract in the future.

2.1.3 Cost of Carry

The previous examples and analysis are based on one important assumption, no transaction costs exist, which is unlike reality, especially in trading the commodity assets. Generally, warehouses charge storage fees; thus, holding an asset costs extra money. Undoubtedly, these costs affect the forward prices. Take a discrete-time model to illustrate the structure of forward price when cost of carry exists. Suppose *T* is the forward expiration date, and *T - 0* is in *M* monthly period. The warehouse charges the monthly storage cost in *c(k)* per unit, and storage costs are paid in advance at the beginning of the month. The forward price is determined by the spot price, discount factor, and storage cost. Hence, the forward price is

$$F = \frac{S}{d(0,M)} + \sum_{k=0}^{M-1} \frac{c(k)}{d(k,M)} .$$

Where

F = Futures price expired at time *T*.

c (k) = Storage cost at time *k*.

d (i, j) = Discount factor, discount from *j* to *i*.

S = Spot price of the forward contract underlying.

This formula also can be written as

$$S = -\sum_{k=0}^{M-1} d(0,k)c(k) + d(0,M)F .$$

2.1.4 The Current Value of a Forward Contract

A forward contract holder may want to know his forward value on and go. The following is a method to achieve it. Suppose a forward contract was written initially with a delivery price of F_0, and the forward price which expires at time *t* is F_t. The current value f_t can be derived by the difference times the discount factor. This value is given by the following statement:

$$f_t = (F_t - F_0)d(t,T)$$

where

d(t, T) = The risk-free discount factor over the period from *t* to *T*.

f_t = The current value of the forward contract.

2.2 Futures

Similar to the forward contract, futures contracts are legally binding standardized agreements on a regulated futures exchange. The futures contract holder is to make or take delivery of a specified product at a fixed date in the future at a price agreed on when the deal is made.

2.2.1 Basic of Futures Contracts

A futures contract is a standardized contract, buying or selling a stipulated quantity and quality, i.e. a certain grade at a set price on a given date in the future. Futures contracts are standardized trading via an exchange; it can be a physical delivery or cash settlement. The exchange helps to define universal prices and insures the counterparty risk. Individuals do not desire to face the counterparty default risk, therefore an exchange as a middleman is in need. Figure 2.2 and 2.3 are the comparison of two processes trading with an exchange and without an exchange.

Category	**Item of Futures**	**The Exchange**
Agricultural Commodity (Food)	Corn, Oats, Soybeans, Soybean meal, Wheat, Barley, Flaxseed, Canola, Rye, Cattle (feeder), Cattle (live), Hogs, Pork bellies, Cocoa, Coffee, Cotton, Orange juice, Sugar, Rice, Lumber	CME, SAFEX.
Metals and Energy	Copper, Aluminum, Gold, Platinum, Palladium, Silver, Crude oil, Heating oil, Gas oil, Natural gas, Gasoline, Propane, CRB index, Electricity, Coal.	CME, ICE, TOCOM, LME, DME, SGX, Nord Pool, UKPX, EEX.

Table 2.1 Categorized commodity futures contracts. Commodity futures contracts can be divided into three categories: agricultural commodity, metals and energy trading in exchanges.

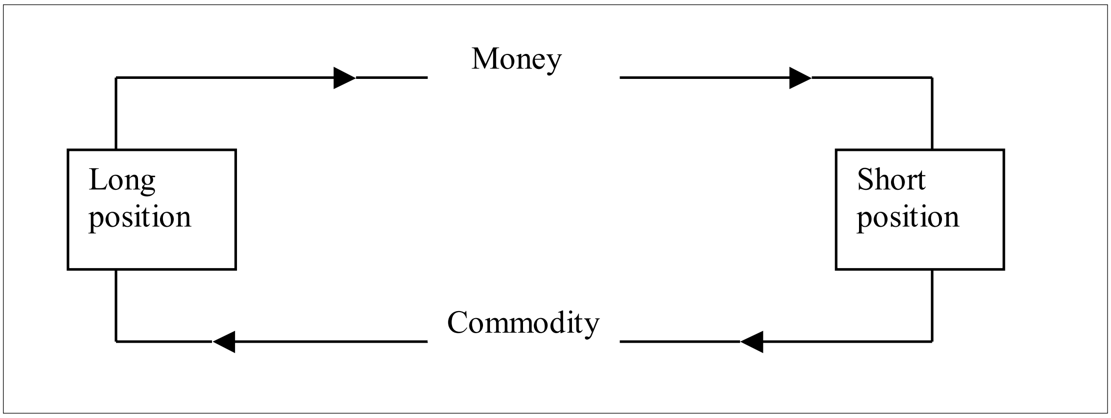


Figure 2.2 Transaction which trades without the exchange (Clearing house). Both sides contact their counterparty directly; therefore both parties encounter more risks if they choose trading via forward markets.

As shown in Fig. 2.2, by trading without an exchange, such as trading via forward contracts, both sides have to face their counterparty default risk directly. Because of that, trading via forward contracts results in both the future price risk and the counterparty risk. On the other hand, as shown in Fig. 2.3, in trading with an exchange, the clearinghouse becomes the direct counterparty for each side- the buyer or seller. The clearinghouse takes both counterparty default risk from doing the opposite position for both buyer and seller. It has the obligation to fulfill the transactions no matter what happen in the other side. For the long side (the buyer), the clearinghouse becomes the seller of the futures contract and vice versa. At the delivery date, the clearinghouse will deliver the commodity to the long position side and receive the commodity from the short side (seller). Consequently, the clearinghouse's position nets to zero. The exchange marked-to-market both sides' position everyday, this ensures both side fulfill their contract in the end.

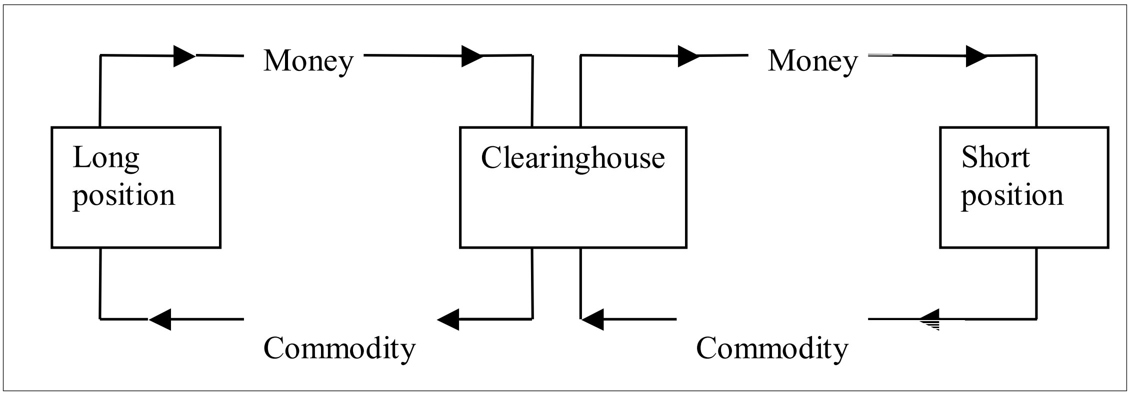


Figure 2.3 Trading with a clearinghouse (an exchange). A decision maker takes less risk trading when buying/selling futures contracts. The future price risk is the only concern the decision maker has to deal with.

The standardization of futures contracts formalizes the delivery dates, quantities per contract, quality (grade) of delivered goods, and even delivery locations. Because transaction makers take less time to build positions, these positions become more liquid and easier to book. For example, when trading a forward contract, the forward price changes constantly. If one does 10 sequence forward contracts, the position builder needs to monitor 10 different contracts thereafter. The futures position terms and conditions can be the same in 10 successive trades; the value will be reflected in the margin account. Also, a trader can offset his futures position from the market.

2.2.2 The Futures Contract and the Margin Account

Futures is a zero sum game, meaning losses and gains com from both side net zero. One person's loss constitutes another person's gain. For the exchange to guarantee the counterparty risk of each side to a minimum level, the margin account is needed. The margin account adjusts its value from market conditions successively; this process is also called marking to market. There are two requirements of a margin account based on daily marked-to-market: initial margin requirements and maintenance margin requirements. If the value of a margin account goes below a defined maintenance margin requirement level (usually about 75% of the initial margin requirement), a margin call is issued to the contract holder asking for additional margin. Otherwise the futures position will be closed out by taking an equal and opposite position. Generally, the margin account is no interest, so some brokers will allow Treasury bills or other securities to composite the account holders' interest.

Here is an example of margin accounts. Supposing one takes a long position (42,000 gallons, 1,000bbls) of heating oil in NYMEX for a March 2010 delivery at a price of \$ 1.8866 (per gallon). And suppose the broker requires a margin of \$800 with a maintenance margin of \$600.The next day the price of this contract drops to \$1.88 and a loss comes out and amount is $-\$.0066\times 42,000 = -\$277.20$. The broker will take this amount from the margin account, leaving a balance of \$522.80. Because \$522.80 is less than 600, the futures holder will be required to deposit \$77.20 in his margin account; otherwise the position will be closed out, so that the contract holder will be forced to realize his loss.

2.2.3 The relationship among Futures Prices, Forward Prices and Spot Prices

This is the same concept as the forward price evaluation— a futures price is influenced by the cost of carry, interest, and spot price and the relationship between spot price and futures price is shown in Fig. 2.4. The spot price of an asset converges to the futures price when the delivery date approaches, otherwise the arbitrage opportunity exists. For example as of today, we all have an expected spot price of the certain underlying after certain period time, *T*. If inequality of the equation happens, $F_T \neq E(S_T)$, the arbitrage opportunity occurs. For example, when there is an inequality, say, $F_T < E(S_T)$, speculators can take advantage from long a position in futures market. At time *T*, one can gain from buying that commodity at F_T and selling the commodity at S_T. The profit from this transaction is $E(S_T) - F_T$. This arbitrage is still valid when the inequality is in the other direction.

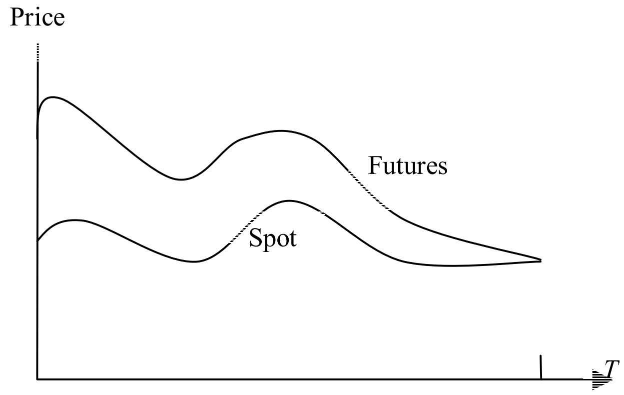


Figure 2.4 Convergence of spot and futures prices.
The futures price converges to the spot price as time approaches the delivery date.

The futures price should be identical with the forward prices. Nevertheless, these two prices are not always the same; the difference between the prices reflects the unmatched cash flows, counter-party risks, liquidity and so forth. On the other hand, these two prices cannot separate too far, otherwise arbitrage will happen.

2.3 Swaps

A swap is an agreement between two parties, both to pay and receive the cash flows at several specified points in the future. It is called swap for ones payment because one is based on the average floating rate during the contract period and the other is based on a fixed rate when the deal is made. Swap is no physical delivery of the any underlying (commodity, equity, fixed income, or FX), normally is net cash settlement. The amount of payment is determined by the agreed terms and conditions. The most typical incentive for the decision maker is to transform the cash flow stream into another type. When traders negotiate the swap, they focus on:

- The fixed price
- The floating price reference
- Pricing period
- Swap period (staring date or effective date, end date or termination date)
- Agreement notional amount

Because the swap can be applied to different markets such as FX, fixed income, commodity, and equity, the value of a swap is derived with regard to the underlying asset. Thus, it is a type of derivatives. The most common is the plain vanilla swap in which one party swaps a series of fixed-level payments for a series of variable-level payments. We will discuss the pricing methodology later.

2.3.1 Plain vanilla interest rate swap

i. Plain vanilla interest rate swap

A plain vanilla swap is when one party receives fixed-level payments and pays a series of variable-level payments. This kind of swap can be regarded as a series of forward/ futures contracts and can be valued by the aggregation of a series of forward/futures. The following is an example of a plain vanilla interest rate swap.

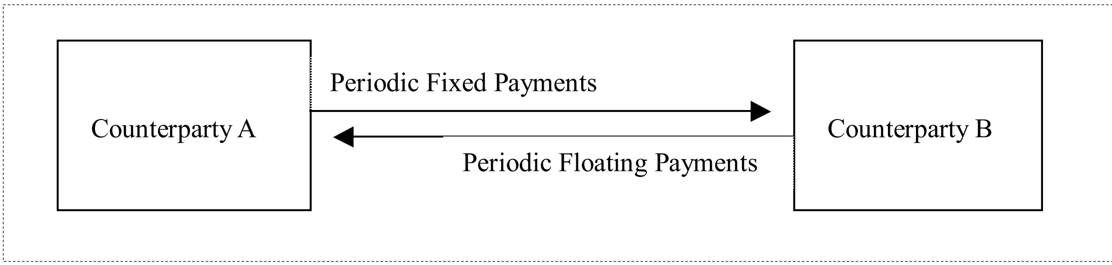


Figure 2.5 Swap is an agreement between two counterparties.

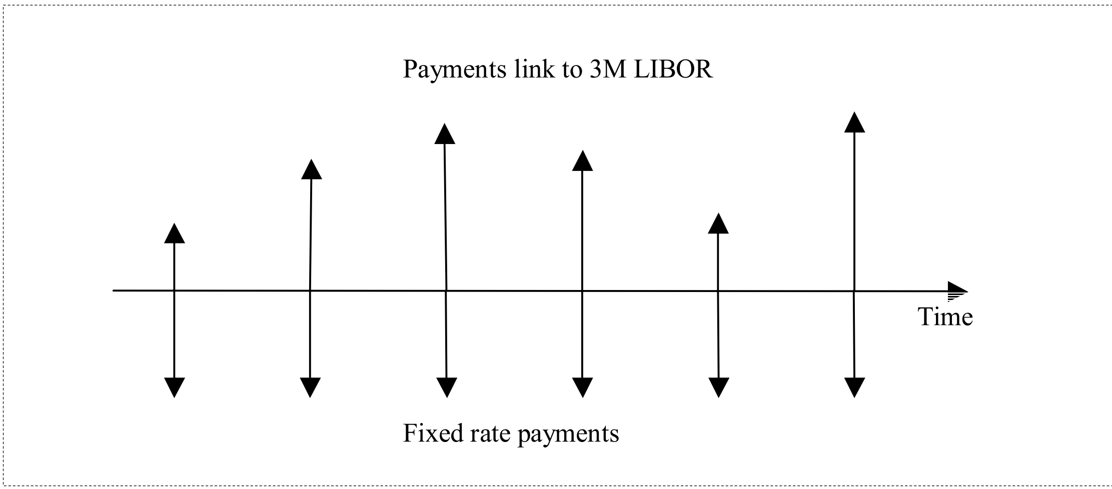


Figure 2.6 Periodic payments of the swap.

Tenor: 3 years

Notional Amount: USD 10,000,000

Payment: Party A pays fixed rate at 3% p.a. and receives 3M USD LIBOR

Payment Frequency: Quarterly

1st Payment result:

If the floating rate is fixed at 3.25%, party A needs to pay party B (3.25% - 3%) X USD 10,000,000/4 = 6,250.

ii. Plain vanilla commodity swap

Swaps are flexible and over-the-counter (OTC), they are tailor-made and customizable transactions, and swaps are financially settled and non-regulated. Therefore, swaps are uniquely suited for hedging applications. These traits make swaps easily applied in energy hedging as well. The most frequently encountered type is the plain vanilla commodity swap, a fixed-price swap

Assuming a power company generates power mainly from natural gas, for the most part, as shown in Fig. 2.7, it purchases natural gas from spot market paying the I*nside FERC's Gas Market Report (IFGMR)* index for each MMBtu delivered. This power company finds the volatility of the IFGMR index is too high and decides to hedge it from the fixed-price swap market. In Fig. 2.8, the power company arranges a pay-fixed swap with a swap counterpart at the same time it pays the spot payment equivalents for the natural gas in the spot market. This power company has eliminated the floating price risk by doing the swap. The net cash flow in each payment date is the fixed payment.

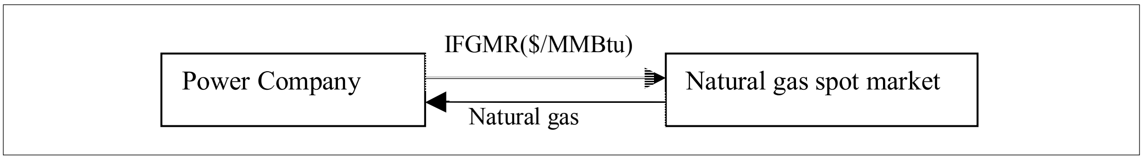


Figure 2.7 Purchasing natural gas from the spot market. The power company normally pays the IFGMR index for natural gas from spot market each MMBtu delivered.

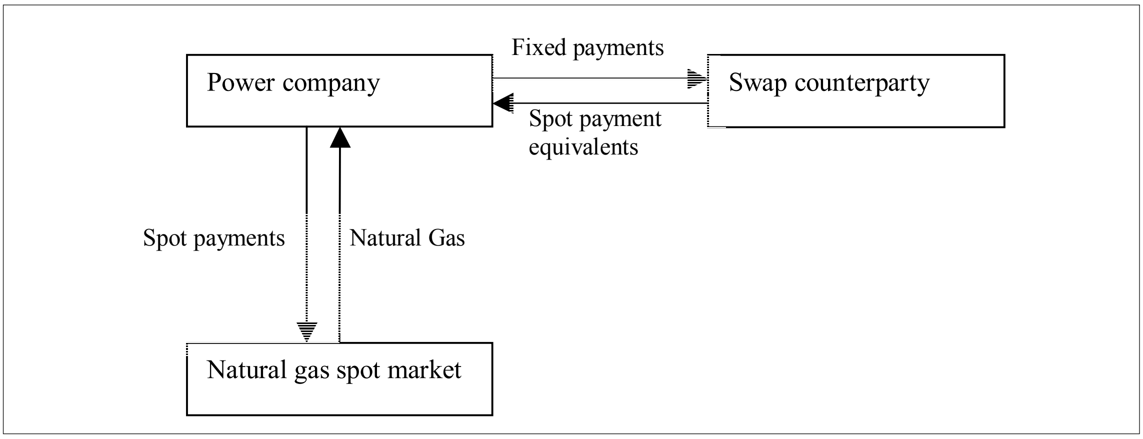


Figure 2.8 Fixed price swap hedge by a power company. The power company buys natural gas on the spot market every month. The company arranges a swap with counterpart (or a swap dealer) to exchange fixed payments for spot price payments. The net effect is that the power company has eliminated the variability of its payments.

2.3.2 Value of a Plain Vanilla Swap

i. Value of a plain vanilla interest rate swap

As in the example earlier, party A agrees to pay a series of semiannual fixed rate payments to party B on a notional principal. Party B pays back a series of quarterly payments based on a floating rate of interest (the example applies current 3-month LIBOR rate) on the same notional principal. Swaps are usually net settled; only the difference of required payments is made by the party that owes the difference. Hence, the swap value can be acquired by summing up the difference during the agreement period. For party A, the swap value is $(F_0 - r, F_1 - r, F_2 - r, \ldots, F_M - r)$ times the principal *N*. The F_i's are the floating rates. When summing up the stream of cash flows, for party A the swap can be seen as processing a floating rate bond and can sell a floating rate bond both of principal *N* and maturity *M*. Hence the overall value of the swap is

$$V = \left[1 - d(0,M) - r\sum_{i=1}^{M} d(0,i)\right]N.$$

Where

V = Party A's swap value.

d(0,M) = Discount factor from period *M* back to initial.

r = Fixed rate party A agreed to initially.

N = Notional amount of the swap.

ii. Value of a plain vanilla commodity swap

A plain vanilla commodity swap can be priced by simply totaling up the cash flows in every payment date. Consider an agreement where party A receives spot price for *N* units of natural gas each period. The net cash flow for this agreement is paying a fixed amount *X* per unit received S_i for *N* units for *M* period is (S_1 - X, S_2 - X, S_3 - X,…., S_M - X) multiplied by the number of units *N*. We get the value of a commodity swap by summing up the discount cash flow stream in each period and the equation is

$$V = \sum_{i=1}^{M} d(0,i)(S_i - X)N.$$

Where

$d(0, i)$ = Discount factor at time zero for cash received at i.

S_i = The spot price for commodity at time i.

X = The fixed price paid for the commodity swap.

N = Number units for the commodity swap.

Generally, X is chosen to make the swap value zero, so that both parties pay nothing in the beginning.

2.4 Options

Unlike the owners of forwards, futures contracts or swaps, Options is an agreement between two parties that buyers only have rights but not obligations to exercise the option. Therefore, at the expiration date, if the market is against the option owner, he would not claim this option. On the other hand, the option seller has only the obligation to exercise the option when the option owner decides to exercise the option, and this only happens when the seller losses money. Options can apply to any asset from a specific futures contract called futures option, a swap called a swaption, or foreign exchange, fixed income, commodity…etc.

2.4.1 Calls (Caps)

Buyers of call options have the right but no obligation to buy the underlying commodity at the specific price, also called the strike price or exercise price, from the call option sellers. Because an option buyer would decide whether to exercise the option, the downside risk he faces is the premium paid. A call option buyer purchases calls only when he thinks the underlying commodity will go up at or before the expiry date. A call option seller sells one's call option right when he thinks the market will go down or run steady before or at the expiration date, so that he can earn the premium—the maximum profit for an option seller.

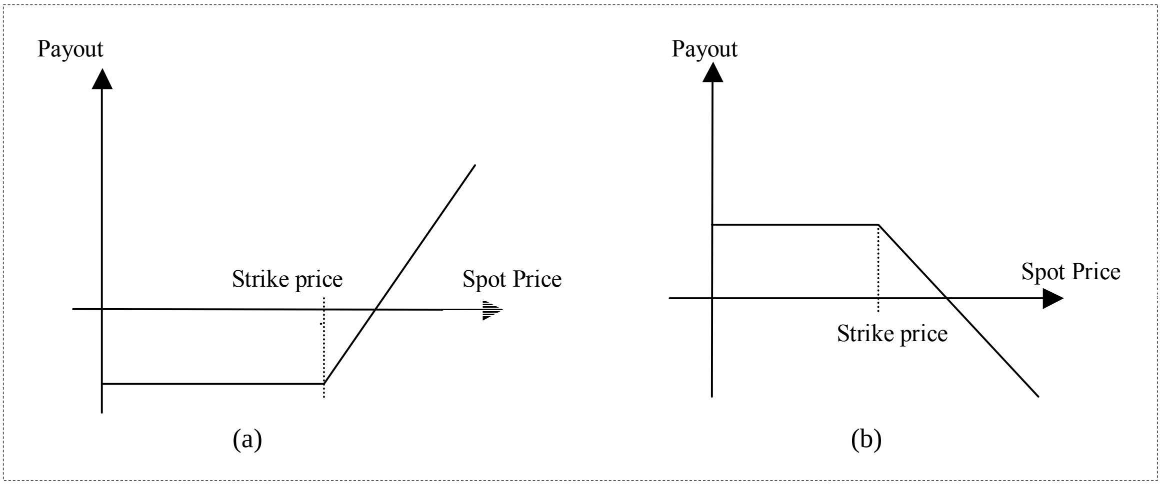


Figure 2.9 Call options. (a) A call option holder has potentially unlimited gains when the commodity price is higher then strike price. Ones maximum loss is the premium the call option holder paid. (b) A call option seller has obligations but not the rights to exercise the option when buyer proposes to claim his rights. The best case scenario for a call option seller is gained from the premium he received.

2.4.2 Puts (Floors)

Call options owner always hope the spot price goes up, On the contrary, the holder of put options only gains when the underlying price is below the strike price. The owner has the rights but not the obligation to sell the underlying commodity at the exercise price. Therefore, the buyer of a put option always projects a bearish market on the underlying.

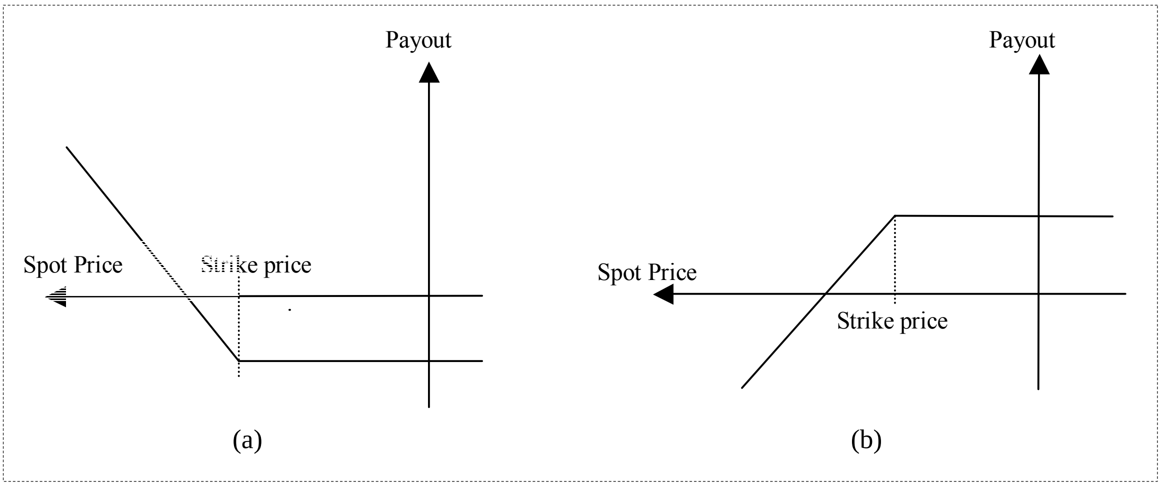


Figure 2.10 Put options. (a) A put option holder gains when the commodity price lower then strike price.and the maximum loss is the premium the option holder paid. (b) A put option seller has obligations but not rights to exercise the option when the buyer proposes to claim his rights. The best scenario for a call option seller is gained from the premium one received.

Options Rights and Obligation	
Call	
Buyer	Has the rights but not obligations to buy the underlying asset at a specific level before or at the expiration date. ***Expectations***: The underlying asset goes up.
Seller	Has the obligations but not rights to sell the underlying asset at a specific level before or at the expiration date. ***Expectations***: The underlying asset steady or goes down.
Put	
Buyer	Has the rights but not obligations to sell the underlying asset at a specific level before or at the expiration date. ***Expectations***: The underlying asset goes down.
Seller	Has the obligations but not right to buy the underlying asset at a specific level before or at the expiration date. ***Expectations***: The underlying asset steady or goes up.

Table 2.2 Options rights and obligations

2.4.3 Option Premium

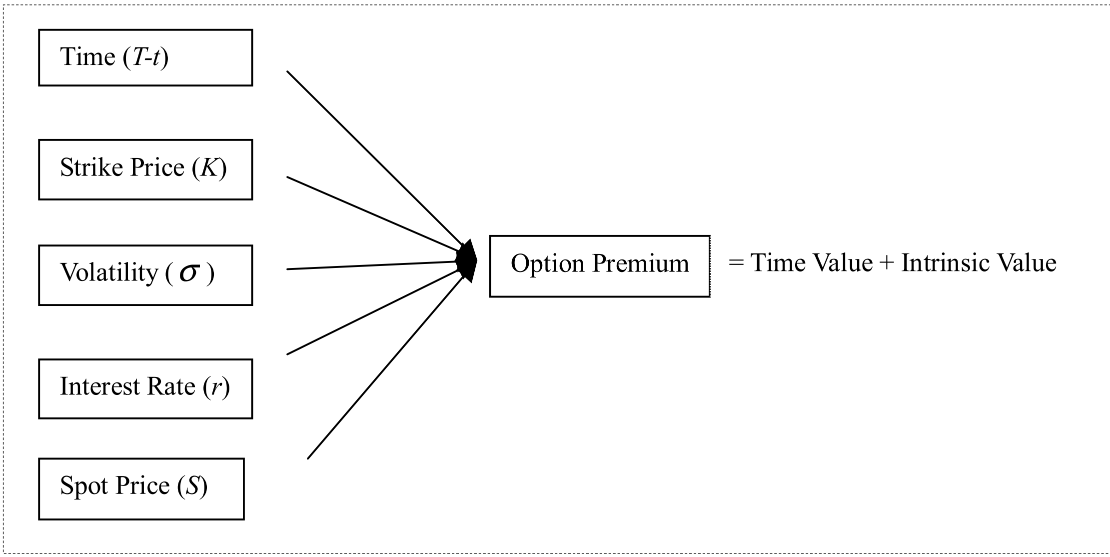


Figure 2.11 Option premium calculation principles. Option prices can compare with each other when these five factors are the same.

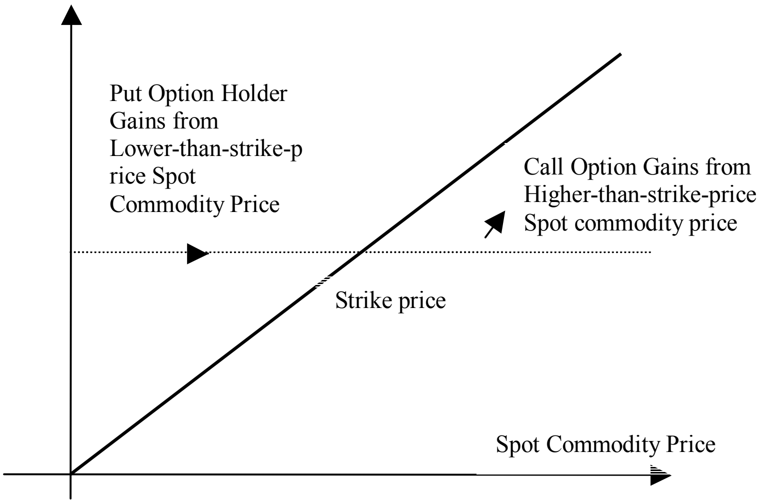


Figure 2.12 Call and Put Options Profits with the Commodity Spot Price,

An Option premium is the cost of the option; it consists of the intrinsic value and time value. The intrinsic value is minimum at zero and larger than zero when the option is in the money, meaning the option will earn money if the spot today remains the same until the expiration date. The time value goes higher as the tenor of the option gets longer; it is of value the option may bring higher profit.

CHAPER 3

HEDGING STRATEGIES

There are two types of hedging, blind hedge and selective hedge. Hedging merely with forward/futures contracts without further planning or quantity analysis is blind hedge; on the other hand, with attentive strategy, planning is selective hedge. The hedging strategies can be simple or complex; the futures contract is the most common derivatives contract eliminating risks for both blind hedge and selective hedge planners. We will illustrate varies hedging strategies later.

3.1 The Perfect Hedge

The simplest and most direct hedging strategy is the perfect hedge**,** where the risk associated with a future commitment to deliver or receive a certain asset can be completely offset by taking an equal and opposite position in one futures market or forward market. By taking the action of buying a forward/futures contract in the opposite direction, or locking in the price of the futures transaction; thus, there is absolutely no price risk. The obligation can only be fully hedged by this strategy when the futures or the forward contract is exactly the same as original position.

A typical example of the perfect hedge is an oil company with an estimated annual capacity of 300,000 barrels of oil who has received a large order for delivery after three months at a specified price. This company finds they can fully hedge their price risk by using WTI traded in NYMEX. They are planning to realize their commitment by exporting 100,000 barrels per month in the consecutive months. The oil company wants to stabilize their profit in advance and decides to hedge by shorting 100 units of WTI futures each month in NYMEX. Assuming the cash flows match timely i.e. both positions will pay and receive the cash flows at the same time, the net effect is that the oil company sells the oil of average of 3-month-WTI futures. If we ignore the slight discrepancy between futures and forwards due to differences in cash flow timing, we can treat the futures contract just like a forward.

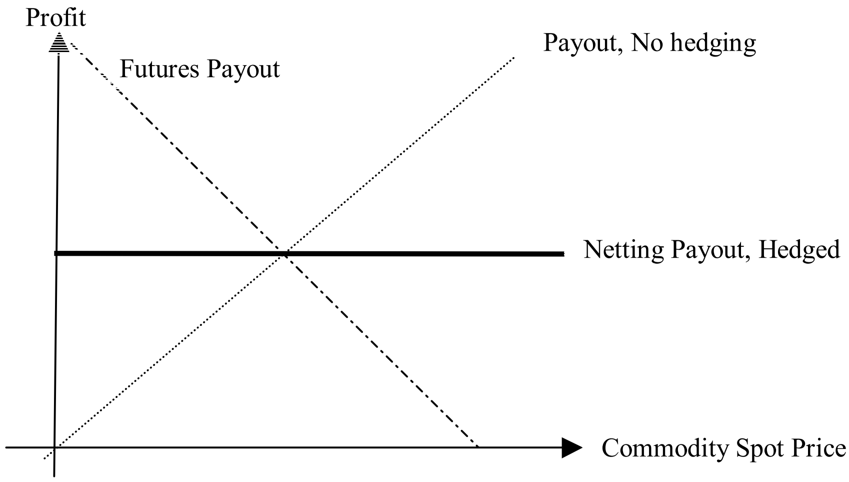


Figure 3.1 The hedged payout. If the oil producer hedges via futures, his payout would become flat; if he doesn't hedge, the payout goes up and down with time.

3.1.1 The Basis Risk that Effects the Perfect Hedge

Since basis is the difference between spot price of asset to be hedged and derivatives underlying price of contract used, basis risk is the risk that price differentiates from the hedged asset and the hedging asset. The follows types of basis risks:

Location basis risk

When the hedging derivatives underlying is delivered in the different geographic region to the physical underlying position, is called location basis risk. In fact different location faced various supply/demand factors, such as political tension, grid problems or in the case of hydrocarbons/gas, pipeline problems, and differ in prices.(Table 3.1)

Quality basis

Though the product is almost the same, say crude oil, the different quality of product cannot be substituted for the product in shortage; for example light sweet WTI and heavy sour Dubai. If one using Dubai futures hedges his light sweet crude oil, he will face this risk.(Table 3.1)

Time basis

Energy/commodity market is a high volatile market; it would be a catastrophe if the price shifts suddenly the large mismatch occurs.

Mixed basis risk

Of course, the mixed basis risk occur when an underlying position is hedged with more than one type of mismatch between underlying asset of hedged and the derivatives. And that hedge is void.

Marker Crudes	Origin/ Benchmark	Quality	Exchange
West Texas Intermediate	USA origin; global benchmark	Light Sweet	NYMEX
Brent	North West Europe, global benchmark	Light Sweet	ICE
Dubai	Middle East origin, Gulf benchmark	Heavy Sour	DME
Tapis	Malaysian origin; Asia Pacific benchmark	Light Sweet	---
Urals	Estern Europe Origin; Mediterranean benchmark	Heavy Sour	RTS

Table 3.1 The specification of crude oil.

3.2 Optimal Hedge

Because the criteria of doing a perfect hedge is not easy to achieve, one of the most main hedging related issues is the determination the optimal hedge ratio. Optimal hedge ratio is derived from optimized the hedging ratio by the particular objective functions, in which the minimum-variance hedging ratio is a widely used strategy (Ederington, 1979; Johnson, 1960; Myers & Thompson, 1989). However, the minimum-variance hedge ratio is based on decision maker whose risk appetite is extremely conservative. Therefore, another hedging strategy, optimum mean-variance hedge ratio, is illustrated in this chapter. In some situation that the risk is nonlinear and can't be reduced easily, this chapter also illustrates this risk in depth.

3.2.1 The Minimum-Variance Hedge

Though perfect hedge is easy and effective, the criteria to hedge perfectly with futures/forward contracts are strict and not always possible. It may be the mismatch from the delivery dates of the futures contract or the amount of the transaction is not divisible by the futures contract number. If the decision maker wants to hedge via forward contracts, he may find the forward contract is illiquid. It is not always possible to construct a perfect hedge with futures contracts. Therefore, most situations reduce the risks instead of eliminating it completely. A widely used measurement of the degree hedging perfectly to the degree of imperfectness is the basis, defined as the mismatch between the spot and futures prices.

Basis = spot price of asset to be hedged – futures price of contract used.

According to the equation above, the best scenario is where the basis is zero at the delivery date. That means the asset price hedged is identical with price of the hedging instrument. However, this situation seldom happens for the reasons mentioned earlier. Hence, alternative hedging techniques are required, one of which is minimum-variance hedge.

3.2.2 The Proof of the Minimum-Variance Hedge

In a world without transaction costs, e.g. interest rate, brokerage fee for margin account, supposed at time zero we would like to hedge our underlying asset by futures expired at time T. At time T we will receive the cash flow, y, consisting of x and hedging profit and loss. x presents the obligation to purchase W units at time T and it equals the spot price of the commodity times the units, WS. It is the spot price of an asset to be hedged at time T. Let F denote the futures price of the contract that is used to hedge. There are two prices related from F, F_T and F_0 (indicating the futures price at time 0 and time T respectively). The overall net cash flow from the whole position is y. Therefore,

$$\text{cash flow} = y = x + (F_T - F_0)h.$$

Where

y = The net cash flow at time T.

F_T = Futures price at time T.

F_0 = Futures price at time 0.

x = The cash flow generated from original transaction equals WS_T.

H = Hedging ratio using minimum-variance hedge method.

To acquire the minimum-variance hedging ratio, we need to define the variance of the cash flow as

$$\begin{aligned}\text{var}(y) &= \text{var}[x + h(F_T - F_0)] \\ &= E\{x + (F_T - F_0)h - E[x + (F_T - F_0)h]\}^2 \\ &= E[x + hF_T - hF_0 - E(x) - hE(F_T) + hF_0]^2 \\ &= E[(x - \bar{x})]^2 + E[(F_T - \overline{F_T})]^2 h^2 + 2E(F_T - \overline{F_T})(x - \bar{x})h \\ &= \text{var}(x) + 2\,\text{cov}(x, F_T)h + \text{var}(F_T)h^2.\end{aligned}$$

Because we want to find the h when the variance is minimized, this requires taking derivative with respect h and equaling zero. This leads to the following result:

$$\frac{\partial\,\text{var}(y)}{\partial h} = 2h\,\text{var}(F_T) + 2\,\text{cov}(x, F_T)$$

$$h\,\text{var}(F_T) = -\text{cov}(x, F_T)$$

$$h = -\frac{\text{cov}(x, F_T)}{\text{var}(F_T)}$$

Using the minimum-variance hedging ratio h applies back to the original cash flow, the resulting variance is

$$\text{var}(y) = \text{var}(x) - \frac{\text{cov}(x, F_T)^2}{\text{var}(F_T)}.$$

When the $X = WS_T$,

$$y = WS + h(F_T - F_0)$$

$$\mathrm{var}(y) = W^2\ \mathrm{var}(S) + h^2\ \mathrm{var}(F_T) + 2Wh\,\mathrm{cov}(S, F_T)$$

$$\frac{\partial\,\mathrm{var}(y)}{\partial h} = 2h\,\mathrm{var}(F_T) + 2W\ \mathrm{cov}(S, F_T)$$

$$h\,\mathrm{var}(F_T) = -W\ \mathrm{cov}(S, F_T)$$

$$h = -W\,\frac{\mathrm{cov}(S_T, F_T)}{\mathrm{var}(F_T)}.$$

and

$$\beta = \frac{\mathrm{cov}(S_T, F_T)}{\mathrm{var}(F_T)}.$$

Therefore,

$$h = -\beta W .$$

The minimum hedge is the best method when the perfect hedge is impossible. In fact, in some special occupations, the minimum-hedge can get the same result as the perfect hedge. Supposing a futures commodity price will be identical to the spot commodity being hedged, $F_T = S_T$. The decision-maker has *the* obligation to buy W units of the commodity, so that $X = WS_T$. In this situation, cov (x, F_T) = cov (S_T, F_T) W = var (F_T) W and we know that h = - W. *Thus,* var (y) = 0, there will be no difference when we use the perfect hedge and the minimum-variance hedge.

3.2.3 Optimal Mean-Variance Hedge Ratio

Viewing the hedging problem from a portfolio perspective is called the optimal hedge. An improvement over minimum-variance hedge is optimal mean-variance hedge, because it incorporated both risk and return in the derivatives of the hedge ratio. Mean-variance hedge ratio satisfies the expected utility maximization principle; however, this method is constrained the utility function to be quadratic. Same as the minimum-variance hedge, assuming that cash flow from the portfolio is x at time T; and supposing that it will be hedged by futures contracts in the h contracts, the optimal hedging ratio could be acquired by the maximized the utility value of the specific utility function.

$$\max_h E[U(y)] = \max_h E\{U[x + h(F_T - F_0)]\}.$$

This approach fully accounts for the specific risk appetite described by quadratic function and varied hedging ratio will be obtained. Assuming ones utility function is

$$U(x) = x - \frac{b}{2}x^2,$$

with $b > 0$.

Where

b = The risk aversion parameter.

x = The cash flow generated from original transaction equals WS_T.

To acquire the optimal hedging ratio easier, we use another equation (close form) which is essentially equivalent to maximizing the expression

$$V(x) = E(x) - r\,\mathrm{var}(x).$$

where

$V(x)$ =An altered mean-variance utility.

r = Positive constant.

In the most cases,

$$r = 1/(2\hat{x}).$$

When $\hat{x}$ is applied by the final value of E(x), weights variance and one-half of $[E(x)]^2$ about equally.

We applied the quadratic utility function into the expected value.

$$Maximize\{E[x + h(F_T - F_0)] - r\,\mathrm{var}(x + hF_T)\}.$$

$$\frac{\partial\{E[x + h(F_T - F_0)] - r\,\mathrm{var}[x + h(F_T - F_0)]\}}{\partial h} = 0$$

$$\frac{\partial\left\{E(x)+h\overline{F_T}-hF_0-r\left[\operatorname{var}(x)+h^2\operatorname{var}(F_T)+2h\operatorname{cov}(F_T,x)\right]\right\}}{\partial h}=0$$

$$h\overline{F_T}-F_0-2rh\operatorname{var}(F_T)+2r\operatorname{cov}(F_T,x)=0$$

After some algebra, the optimum mean-variance hedge ratio is

$$h=\frac{\overline{F}_T-F_0}{2r\operatorname{var}(F_T)}-\frac{\operatorname{cov}(x,F_T)}{\operatorname{var}(F_T)}.$$

3.4 Hedging Nonlinear Risk

In may be the case that the payout function is not changing along with the underlying price linearly. This happens in cases where the final wealth *x* was a linear function of an underlying market variable, e.g. the commodity price. Disappointingly, nonlinear risk cannot be hedged perfectly by derivatives, which we will illustrate it in this section.

Nonlinear risk can arise in complex contracts as well; such as when a U.S. oil company is drafting a future agreement to sell crude oil to a Norwegian company in Kroner at a future date at a specified price. Since the U.S. firm takes a large foreign exchange price risk, both parties agree to share that risk if Norwegian Kroner (NK) appreciates or depreciates more that 10%. Below that level, the U.S firm alone would absorb the risk. To the U.S. firm, nonlinear risks also arise when the price of a good is influenced by other factors like the quantity being bought or sold, because oil prices change with the demand and supply instantaneously. When the oil supply in that month is a surplus, oil prices decline. In other words, production amount is another factor that affects the market price.

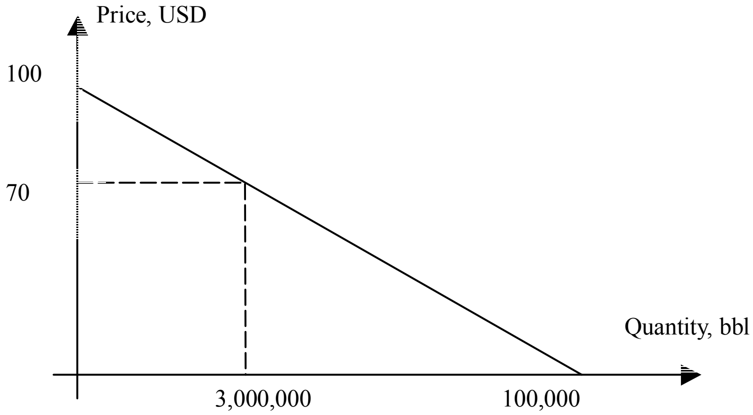


Figure 3.2 Demand for crude oil. The price of crude oil varies from $100 to $0 per barrel, depending on the total quantity produced.

As an oil product, summer production in Mexico Gulf depends on the weather. If no hurricane attacks during the season, it can drill oil in full-capacity. Otherwise, capacity depends on the number of days of attack and the strength of the hurricane. All produced oil is harvested simultaneously, and the price per barrel is determined by a market demand function, which is shown in Fig. 3.2. This demand function is

$$P = 100 - D/100{,}000.$$

where

D = The demand amount among the crude oil market. Each oil producer will produce an amount of crude C which is not uniform.

P = The crude market price under demand and supply equilibrium.

Assuming that the amount of oil drilled on each field can vary, between 0 and 60,000 barrels, with expected value $\overline{C}$ = 30,000 and there are a total of 100 fields, and thus $\overline{D} = 3{,}000{,}000$. A producer will have his revenue by

$$R = PC = \left(100 - \frac{D}{100{,}000}\right)C = 100C - \frac{C^2}{1{,}000}.$$

Where

R = Revenue by selling crude oil.

C = Each oil producers' production amount.

This shows that the revenue is a nonlinear function of the underlying uncertain variable *C*. This means, for the oil producer, its risk is not one factor but two factors. In other words, the oil producer faces nonlinear risk.

In this complicate situation, the price is uncertain until oil has been produced; the oil producer only reduces partial risk by hedging with futures contracts when the producer can predict his production in the futures. Thus nonlinear risk cannot be reduced completely by plain vanilla derivatives.

CHAPTER 4
UTILITY FUNCTIONS AND OPTIMAL HEDGING RATIO

4.1 Utility Function
4.1.1 Introduction

Fundamentally, there are two ways to evaluate a random cash flow: (1) directly, using measures such as expected value and variance; and (2) indirectly, by reducing the flow to a combination of other flows which have already been evaluated. This chapter focuses on these two approaches, showing how they apply to single-period investment problems—and showing how they work together to produce strong and useful pricing relationships.

4.1.2 Utility Functions

The utility function will have two properties: (1) It will be order preserving. In other words if we measure the utility of x as greater than the utility of *y*, *U(x)* > *U(y)*, it means that *x* is actually preferred to *y*. (2) Expected utility can be used to rank combinations of risky alternatives. And we can write the expected utility of wealth as follows:

$$E[U(W)] = \sum_i p_i U(W_i)$$

where

W = The amount of wealth.

$U(W)$ = The utility function of wealth amount.

p_i = The probability of outcome in *i* event.

Given the five axioms of rational investor behavior and the additional assumption that all investors always prefer more wealth to less, we can say that investors will always seek to maximize their expected utility of wealth. In fact, the above equation is exactly what we mean by the theory of choice. All investors will use it as their objective function. In other words, they will seem to calculate the expected utility of wealth for all possible alternative choices and then choose the outcome which maximizes their expected utility of wealth.

4.1.3 Three Types of Utility Functions

An important thing to keep in mind is that utility functions are specific to individuals. There is no way to compare one individual's utility function to another's. A useful way is to identify three simple utility functions which assume that more wealth is preferred to less, in other words, the marginal utility of wealth is positive (*MU(W) > 0*). If *x* and *y* are positive numbers, and *x > y*, then *U(x) > U(y)*. In general, if the utility of expected wealth is greater than the expected utility of wealth, the individual will be risk averse.

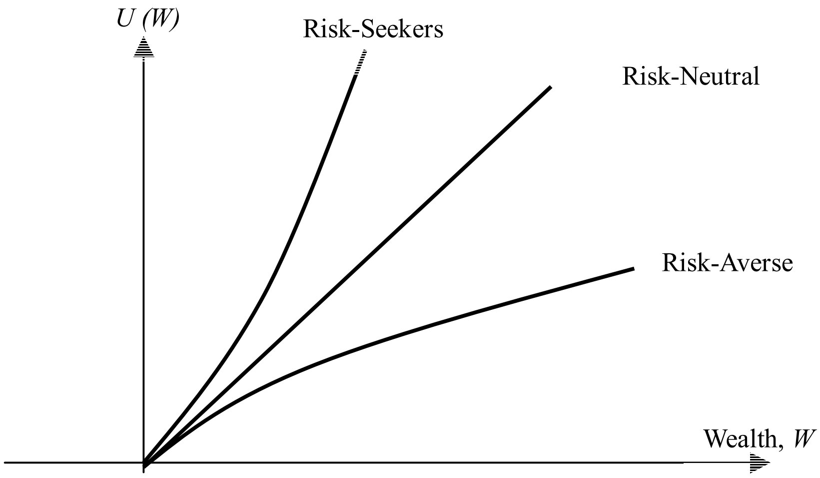


Figure 4.1 Three types of utility functions: Risk aversion, risk neutrality, risk seeking. If $U[E(W)] > E[U(W)]$,we have risk aversion. If $U[E(W)] = E[U(W)]$,we have risk neutrality. If $U[E(W)] < E[U(W)]$,we have risk seeking.

If $U[E(W)] > E[U(W)]$, the utility function is strictly concave, this is risk aversion. The decision maker will get more risk increased premium when taking one more unit of risk. If $U[E(W)] = E[U(W)]$, the utility function is linear, that is risk neutral which means the decision-maker will not get extra risk premium to take another unit of risky position. $U[E(W)] < E[U(W)]$, the utility is convex, this is risk seeking; among the players in the financial market, financial institution and hedge fund are risk seekers because they are so eager for return they ask for extra risk premium in turn.

There is another method to differentiate risk appetite: supposing a man can make a choice, either making an investment which may receive x dollars or nothing in the future; nor keep his $ 100 in cash. Assuming the expected value of this investment is USD 100; the man who puts the money into uncertainty is a risk lover, the one who is indifferent to the two choices is risk neutral; and one who prefers keeping money in his pocket is a risk averter.

4.1.4 The Common Types of Utility Functions

Individuals possess specific utility functions, depending on their individual risk tolerance and individual financial condition. The simplest utility function is the linear utility function, *U (W) = W*. Except for the fact that utility is a function with the amount of wealth, this utility function is only for the risk neutral. Other types of utility function models acquire more condition to discern risk types. The four most common types of the utility function as followed:

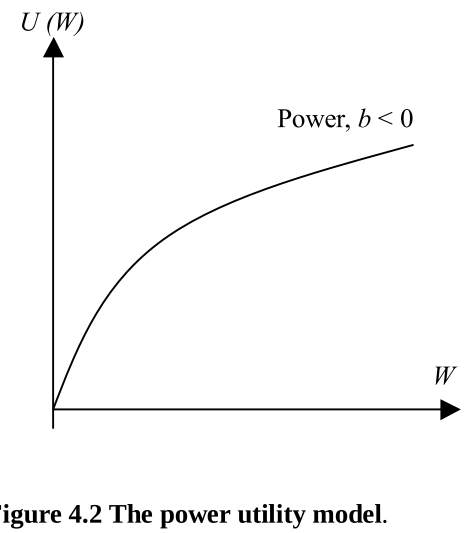


Figure 4.2 The power utility model.

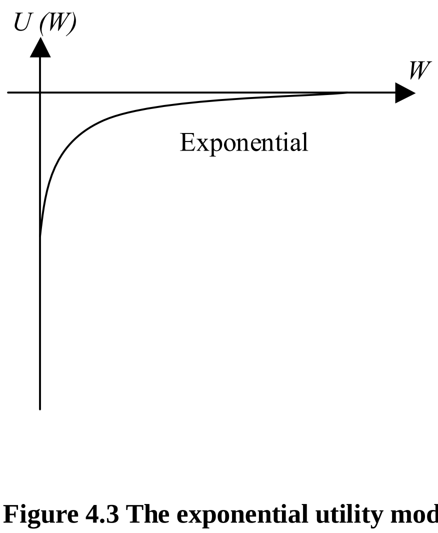


Figure 4.3 The exponential utility model.

i. Power Utility Function Model

$$U(W) = bW^b$$

Where W = The amount of wealth, b is the parameter in charge of the degree of the concaveness, $b > 0$ is a risk lover, $b = 0$ is a risk neutral, and $b < 0$ is a risk averter (Fig. 4.2).

ii. Exponential Utility Model

$$U(W) = -e^{-aW}$$

An exponential utility model may come with a negative value when $a > 0$ as shown in Fig. 4.3; its value may be negative. Under this situation, exponential utility model is still reasonable, since the utility number itself is meaningless rather than a relative value. Under the exponential utility model, U (W) is capped at 0 and the minimum value is where W equals to zero.

iii. Logarithmic Utility Model

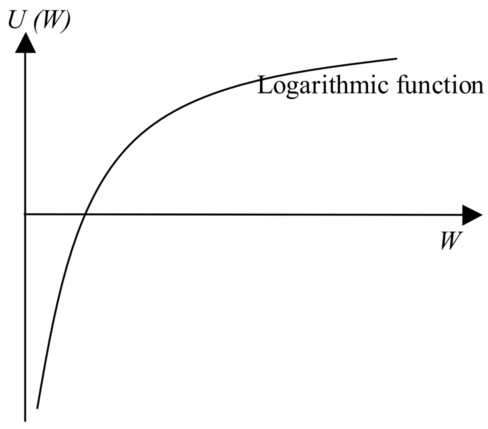


Figure 4.4 The logarithmic utility model. When the utility function is a logarithmic function, the utility value can be zero and the minimum value is $-\infty$.

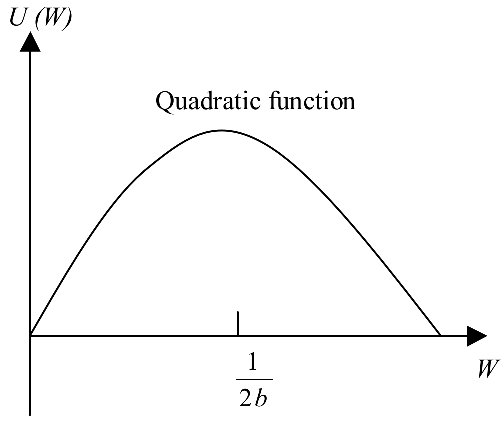


Figure 4.5 The quadratic utility model. The utility value increases as *W*, wealth amount, increased. At the point *U (W)=1/2b* is where maximum utility.

As Fig. 4.4, the Logarithmic utility function is

$$U(W) = \ln(W),$$

it is the function of wealth amount. Under this model, negative values still appear even if the $W > 0$; at the point $W = 0$, the least value is $-\infty$.

iv. Quadratic Utility Model

$$U(W) = W - bW^2$$

Where b is the positive parameter of the utility function; the quadratic utility function only has meaning when $\frac{1}{W} > b$, because within this range, the utility is increasing as increasing of wealth amount (Fig. 4.5). The maximum utility appears when $W = \frac{1}{2b}$.

4.2 Value of Information

One of the important notions of efficient capital markets depends on the quality of information and value of information. A message about likelihood over a period of time related to different events is defined as the information structure. The message has value to whomever (1) can react—take action, or change his/her previous decision based on the message, in other words, can change previous behavior and (2) will benefit in wealth or increased utility payoff from that reaction. Take an easy example, "the hurricane may attack Mexico Gulf within 7 days" only can be of value to the oil producers who have oil drillers in Mexico Gulf and are able to evacuate to prevent losses. For one who has invested nothing in that area or cannot withdraw his well equipments, that information is worthless. A formal expression of the above concept defines the value of an information structure, $V(\eta)$, as

$$V(\eta) \equiv \sum_{m} q(m) \underset{a}{MAX} \sum_{e} p(e \mid m) U(a,e) ,$$

where

q(m) = The marginal probability of receiving a message *m*.

p(e|m) = The conditional probability of an event *e* given a message *m*.

U(a,e) = The utility resulting from an action *a* if an event *e* occurs. We will call this a benefit function.

According to the equation above, a decision-maker will evaluate an information structure, which considers all possible outcomes and possibilities and chooses an action which will maximize the expected utility given a received message. For each possible message we can determine the optimal action. Mathematically, this process can be presented in a formula below

$$\underset{a}{MAX} \sum_{e} p(e \mid m) U(a,e) .$$

The final step of getting the information value is by weighting the expected utility of each optimal action by using the probability, *q(m);* this is the probability that an action will be made when receiving the message m_i. This way the decision maker knows the expected utility of the entire set of messages, which we call the expected utility (or utility value) of an information set, $V(\eta)$.

An Example of Value of Information

The following example applies the value-of-information (VOI) concept to portfolio assets allocating. Assuming we can either put money in risk-free assets with an annual return of 3%, or invest money in the commodity market with an expected return of one of three possible results, 16%, or 10%, or -5% (as Figure 4.6), a decision–maker will build his portfolio in combination of two assets in a specific distribution. Assuming the standard deviation of the commodity market, σ_m, is known with certainty.

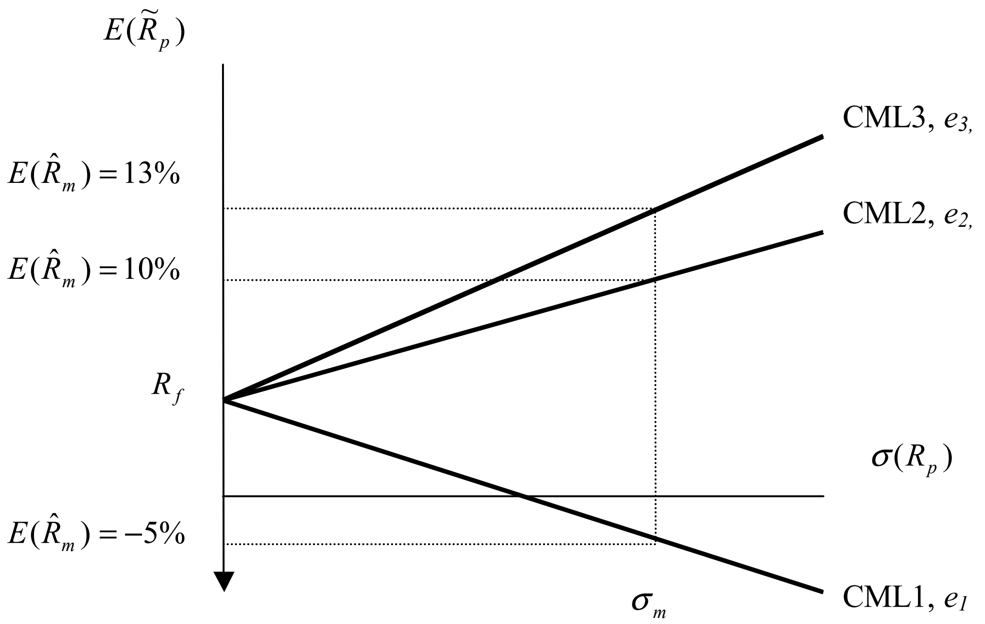
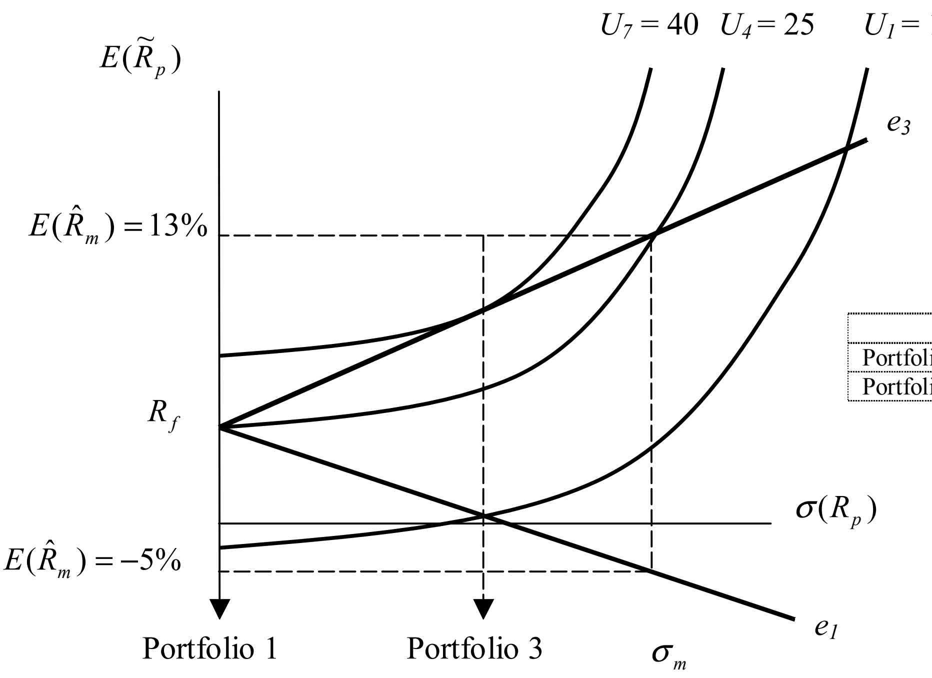


Figure 4.6 Three capital market lines depict the possible outcomes. Based on three possible returns and a known standard deviation, σ_m, on the commodity market build three linear efficient sets (the capital market lines, CML*i*).

Under this framework, every rational investor chooses an optimal portfolio where ones indifferent curve is tangent to the efficient set along the CML_i— each possible outcome. In this example, the decision maker is a risk averter, because the utility curve slope is increased when one more unit risk is added. In other words, the investor will get more risk premium for increased risk.



	% in R_f	% in R_m
Portfolio 1	100	0
Portfolio 3	30	70

Figure 4.7 Optimal choices for two future states, e_1 or e_3, of the world.

Benefit Function

From the formula mentioned earlier to calculate the value of an information, we need to establish the utility payoff function (benefit function), *U(a,e)*. *U(a,e)* is the utility function when taking a course of action, *a*, under an event or a state of the world, *e*, occurs. Decision-maker can put the portfolio in a_1 = portfolio 1, a_2 = portfolio 2, and a_3 = portfolio 3 in three states of the world by a particular market return: e_3=13%, e_2= 10%, and e_1= -5%.

Presuming the commodity market return is 13%; a decision maker allocates funds in the distribution of portfolio 3 and if the commodity market return goes as expected, the utility payoff will be U_7 = 40. If an accident occurs, commodity return turns out to be -5%. However, the decision maker has already put money in portfolio 3; the utility payoff is where the utility curve tangents CML 1, U_1, minus the disappointment, U_4-U_1,. Thus, the utility amount is U_1-(U_4-U_1) = -5.

Action	***e_1 (R_m= -5%)***	***e_2 (R_m= 10%)***	***e_3 (R_m= 13%)***
Portfolio 1 (action a_1)	$U_4 = 25$	$2U_4 - U_5 = 20$	$2U_4 - U_7 = 10$
Portfolio 2 (action a_2)	$2U_2 - U_4 = 15$	$U_5 = 30$	$2U_6 - U_7 = 26$
Portfolio 3 (action a_3)	$2U_1 - U_4 = -5$	$2U_3 - U_5 = 16$	$U_7 = 40$

Table 4.1 Benefit function *U(a,e).*

Information Structure

Information structure is another essential element for calculating the value of information; a Markov matrix gives the probability that an event will actually occur given that a particular message has been received. Obviously, two totally opposite cases are perfect information and no information. In table 4.1, a world with perfect information means the message predicts the result perfectly. This way the decision-maker always makes decision with maximum utility. And, information has its greatest value in a perfect information world.

η_2 = ***perfect information***				η_0 = ***no information***				η_1 = ***noisy information***		
	m_1	m_2	m_3		m_1	m_2	m_3		m_1	m_2	m_3
e_1	1.0	0	0	e_1	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	e_1	.6	.3	.1
e_2	0	1.0	0	e_2	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	e_2	.2	.5	.3
e_3	0	0	1.0	e_3	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	e_3	.2	.2	.6

Table 4.2 Information structures.

When regarding a received message providing the estimation probably of future states of the commodity market return, the decision maker will alter the original decision by choosing another action where he can maximize expected utility given that message.

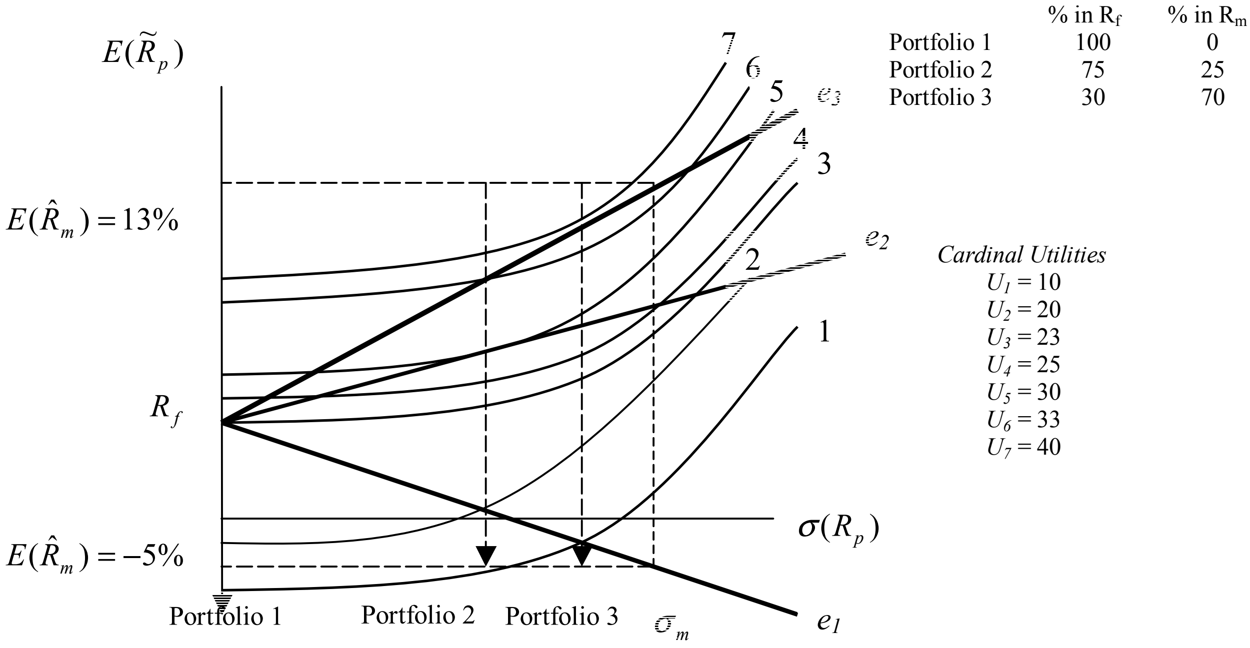


Figure 4.8 Optimal actions given various states of world.

Perfect Information, η_2

In a perfect information world, when receiving $\boldsymbol{m_1}$, we know the state of the world e_1 will obey with certainty. As the result, we will take action under each message obtained as followed.

Message	***Optimal action***	***Utility***
$\boldsymbol{m_1}$	a_1 (invest in portfolio 1)	25
$\boldsymbol{m_2}$	a_2 (invest in portfolio 2)	30
$\boldsymbol{m_3}$	a_3 (invest in portfolio 3)	40

Each message is given in even possibility $q(m_1)$ =1/3, $q(m_2)$ =1/3, and $q(m_3)$ =1/3. In other words, each action is made in equal possibility. The utility value of perfect information is

$$V(\eta_2) = \frac{1}{3}(25) + \frac{1}{3}(30) + \frac{1}{3}(40) = 31.667 .$$

To calculate the value of information, we need to transfer the utility into dollar amounts. Assuming we have the utility function in

$$U(W) = 10\ln(W - \$200).$$

$$31.667 = 10\ln(W(\eta_2) - \$200)$$
$$3.1667 = \ln(W(\eta_2) - \$200)$$
$$\$23.7290 = W(\eta_2) - \$200$$
$$\$223.7290 = W(\eta_2)$$

From the utility function, we derived the equivalent amount of wealth under perfect information world to be $223.7290.

No Information, η_0

In the example of a no information world— the accuracy of every message is the same, $\frac{1}{3}$. In other words, no matter which message we received, three states of future still happen equally.

Since each message and future state are independent, the possibilities of outcomes are identical. Regardless of which message has been received, the decision-maker always takes a_2 as his optimal decision because it generates the highest expected utility.

If we take action	***The expected utility is***
a_1 (invest in portfolio 1)	$\frac{1}{3}(25) + \frac{1}{3}(20) + \frac{1}{3}(10) = 18.33$
a_2 (invest in portfolio 2)	$\frac{1}{3}(15) + \frac{1}{3}(30) + \frac{1}{3}(26) = 23.67$
a_3 (invest in portfolio 3)	$\frac{1}{3}(-5) + \frac{1}{3}(16) + \frac{1}{3}(40) = 17$

As the result, the utility value of η_0 is

$$V(\eta_0) = \frac{1}{3}(23.67) + \frac{1}{3}(23.67) + \frac{1}{3}(23.67) = 23.67\,.$$

To convert the utility amount to the wealth amount:

$$U(W) = 10\ln(W - \$200)\,.$$

$$23.67 = 10\ln(W(\eta_0) - \$200)$$

$$2.367 = \ln(W(\eta_0) - \$200)$$

$$\$10.6653 = W(\eta_0) - \$200$$

$$\$210.6653 = W(\eta_0)$$

Using the utility value above, we can get the value of perfect information in the dollar amount to be

$$W(\eta_2) - W(\eta_0) = \$13.1727\,.$$

Noisy Information, η_1

In the case of noisy information, messages received are random. The preciseness of noisy information messages is in the middle of no information and perfect information. For example, Table 4.2 shows when $\boldsymbol{m_1}$ is received, there is only 60% probability that first state of world might be obtained; and 40% probability the message might be wrong, 20% in e_2 and 20% in e_3 state. According to the information structure, a decision-maker has three options (investing in portfolio 1, 2, or 3 individually) when each received message and the expected utility is as followed:

When received $\boldsymbol{m_1}$,

If we take action	***The expected utility is***
a_1 (invest in portfolio 1)	***.6 (25) + .2 (20) + .2 (10) = 21***
a_2 (invest in portfolio 2)	.6 (14) + .2 (30) + .2 (26) = 20.2
a_3 (invest in portfolio 3)	.6 (-5) + .2 (16) + .2 (40) = 8.2

The decision-maker will take a_1 as his action, because it has the largest expected utility amount.

When received $\boldsymbol{m_2}$, and the optimal action is a_2.

If we take action	*The expected utility is*
a_1 (invest in portfolio 1)	.3 (25) + .5 (20) + .2 (10) = 18.5
a_2 (invest in portfolio 2)	***.3 (14) + .5 (30) + .2 (26) = 24.4***
a_3 (invest in portfolio 3)	.3 (-5) + .5 (16) + .2 (40) = 14.5

When received $\boldsymbol{m_3}$, and the optimal action is a_3.

If we take action	*The expected utility is*
a_1 (invest in portfolio 1)	.1 (25) + .3 (20) + .6 (10) = 18.5
a_2 (invest in portfolio 2)	.1 (14) + .3 (30) + .6 (26) = 26
a_3 (invest in portfolio 3)	***.1 (-5) + .3 (16) + .6 (40) = 28.3***

As the result, the decision-maker would choose his optimal action when receiving each message as follows:

If we receive	*The optimal action is*	*With expected utility*
$\boldsymbol{m_1}$	a_1	21
$\boldsymbol{m_2}$	a_2	24.4
$\boldsymbol{m_3}$	a_3	28.3

$$V(\eta_1) = \frac{1}{3}(21) + \frac{1}{3}(24.4) + \frac{1}{3}(28.3) = 24.57$$

If the corresponding dollar value, for the *i*th individual, is $W(\eta_1) = \$211.6697$, then the noisy information value is 211.6697 - 210.6653 = 1.0044.

Conclusion

A seemly hedging strategy can't be made arbitrarily, the decision maker must always put a lot of effort into it— collecting information, distinguishing valuable information, making decisions, and selecting suitable tools— which cost time and money.

This thesis presents a review of hedging tools in energy futures, forward, swap, and option markets. It is intended to illustrate a beneficial method for decision markers to initiate a proper hedging strategy or an intuitive sense of how it works. This thesis begins with a review of basic tools for hedging and offers all the hedging strategies that can be used ranging from perfect hedging, to minimum-variance hedging and optimal hedging, to hedging the nonlinear risk. As we know, the perfect hedge is the best, since it reduces the price risk completely. Nevertheless, there are not many occasions on which the perfect hedge is applied, because there is not always a market exists of its restrictions and the rarity of the market or the underlying fitting. Finding the alternative is necessary for decision makers.

The final part of the thesis reviews the three types of decision maker's risk preference and the methods of valuing information. According to the precious information, recognized individual's risk preference, hedging transactions should reduce high volatility of earning or cost in the energy industry.

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